Homogenization of both linear and nonlinear highly heterogeneous plate

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Composite plates are widely used in aeronautics applications because they offer excellent ratio between stiffness or strength performance and weight. The size of fine scale details in these heterogeneous plates is typically much smaller compared to the dimensions of the structure, thus making direct numerical analyses is prohibitively expensive. To avoid these large-scale computations, it is preferable to model these plates at the macroscale as a homogeneous continuum with effective properties obtained through a homogenization procedure. Based on asymptotic homogenization concepts (Caillerie, 1984 ; Kohn and Vogelius, 1984) discussed the homogenization of heterogeneous periodic linear elastic plates. These models are mathematically elegant and rigorous but only related to a simple engineering model (the Kirchhoff plate model). The Kirchhoff–Love plate model is the simplest and the most widely-used theory. Nevertheless this model neglects the contribution of out-of-plane stress components to the stress energy. However, when the plate slenderness ratio L/h (h is the plate thickness and L is the characteristic dimension of its mid-plane) decreases, out-of-plane stresses have an increasing influence on the plate deflection. Exactly as (Cecchi and Sab, 2007) did for Reissner-Mindlin homogenization of periodic plates, Lebée and Sab, 2012 propose a homogenization theory for their bending gradient theory (Lebée and Sab, 2011). **This method of homogenization concerns only the linear case and may be numerically implemented in order to compare this new and recent developments to our approach which is also valid in linear case (see below).** This approach correct the homogenization theory of Lewinski, 1991; Caillerie, 1984 ; Kohn and Vogelius, 1984 in order to take into account of out-of-plane stress components (transverse shearing and transverse normal stress). So they use implicitly the superposition principle and then theses theories are limited to linear elasticity.

The study of Petracca *et al*., 2017, focused on periodic brick-masonry walls, the macro-scale behavior obeys a Reissner-Mindlin and the local heterogeneous structures is assumed to be transverse isotropic. For macroscopic Reissner-Mindlin plate model, Terada *et al*., 2017 propose a new numerical plate testing (NPT) by adding a specific microscopic displacement terms such that the out-of-plane microscopic shear strain components, contain the macroscopic curvature associated with torsional deformation.

The method of homogenization proposed by Lee, *et al*., 2014 is deduced from the introduction of the double scale asympytotic expansion method into a new double scale variational formulation. The developments given for this method is valid for the macro-scale Reissner–Mindlin behavior and restricted to small deformations and large rotations and displacements. As a consequence, this approach seems to be intractable in the fully nonlinear setting. This is due to fact that the simplification linked to the application of variational-asymptotic method (Berdichevsky, 1979 ; Sutyrin, 1997) runs only under the small deformation assumption. Moreover the recent extension of the asymptotic expansion for homogenization of plate made of non linear Saint Venant-Kirchoff proposed by Kalamkarov *et al*., 2017 seems to be restricted to bending and stretching. **Theoritical and numerical studies of this approach will be investigated in this phd thesis.**

Also the work of this phd thesis concerns both theoritical and numerical studies of Reissner-Mindlin macroscopic plate (the macroscopic displacement field is assumed to be of Reissner-Mindlin type). So we avoid the Saint Venant-Kirchoff asumption considerd in Coenen *et al.,* 2010; Cong *et al.,* 2015. The mechanical behaviour of the constituents of the plate is of non linear hyperelastic type. Then it is necessary to define the relation between the definitions of the macroscopic generalized strains and stresses for a plate continuum in terms of the microscopic ones. This macro-to-micro scale transition is performed by imposing the macroscopic generalized deformation gradient on the RVE (representative volume element) through the essential boundary conditions that may be periodic conditions. Upon solution of the microstructural boundary value problem, the macroscopic generalized stress resultants are expressed by averaging the computed RVE stress field through the use of a generalised Hill-Mandel condition for shell (i.e. an energy condition where the energy in the macroscale is equal to the one in the microscale). In our work, the through thickness dimension is directly combined with the in-plane homogenization.

Also he can be implement numerically the new theoritical model bidimensional which valid for heterogeneous plate proposed by Pruchnicki (2019) (which does not include homogenization concept) which is an extension of a new type of bidimensional model for homogeneous plate (Schneider *et al.* 2014).

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