PhD at University of Lille
Department of mathematic mechanical section

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<u>Period:</u> 01 09 18 - 01 09 21
<u>Scholarship</u>: 1500 per months
Possible Co-tutelle with Politecnico di Torino

<u>Subject :</u>

Homogenization of both linear and nonlinear highly heterogeneous plate and shell

From now the theory of homogenization for highly heterogeneous three dimensional structures is well established. It is not the same for theory of homogenization of highly heterogeneous thin structure (beam, plate or shell). Fom now the best mathematical theory is presented by the well known theory of Caillerie-Kohn-Vogelius. These models are mathematically elegant and rigorous but only related to a simple engineering model (the Kirchhoff plate model). A rigourous mathematical extension to higher order plate model has been proposed by Pruchnicki (2017a) but this model is linked to a problem of scaling of the dimensions of the representative volume element (RVE) as discussed previously by Lewinski (1991) (The geometrical shape of the VER is deformed in the asymptotic analysis). But the most important problem results from *a priori* scalings, assuming applied loads (or deformation) be of certain orders of shell thickness. In reality applied loads are external, which should not be directly related to the thickness in various situations. Results for homogeneous thin structures suffer from the same drawback.
To overcome these difficulties, the recent work of Pruchnicki 2018 (a, b) propose a multiscale finite-strain shell theory for simulating the mechanical response of highly heterogeneous shell with varying thickness (in both linear and non linear cases). To resolve this issue a higher-order stress-resultant shell formulation based on multiscale homogenization is considered. At the macroscopic scale level, we approximate the displacement field by a fourth-order Taylor-Young expansion in thickness. We take account of the microscale fluctuations by introducing a boundary value problem over the domain of a three-dimensional representative volume element (RVE). For the sake of simplicity, the microstructure is assumed to be periodic with respect to curvilinear coordinates. The RVE is subjected to body force and stress vectors acting on both the upper and lower faces. Then this load act obviously on the macroscopic through the local scale. The geometrical form and the dimensions of the RVE are determined by the representative microstructure of the heterogeneity. In this way, an in-plane homogenization is directly combined with a through thickness stress integration. As a result the macroscopic stress resultants are obtained from microscopic stress through a specific form the macro-micro Hill-Mandel condition (which expresses the equivalence between both the internal macroscopic and microscopic energies). All microstructural constituents are modeled as first-order continua and three-dimensional continuum, described by the standard equilibrium and the constitutive equations. This

type of theory is anxiously awaited. The aim of the thesis is to numerically implement this new theory and then to investigate resulting numerical test.

Finally the student can be implement numerically the new theoritical model bidimensional which valid for heterogeneous plate proposed by Pruchnicki (2017b) (which does not include homogenization concept) which is an extension of a new type of bidimensional model for homogeneous plate (Schneider *et al*. 2014).

Pruchnicki, E. (2017 a). Homogenization of a second order plate model. Mathematic and Mechanic of solid, DOI: 10.1177/1081286517719939.

Lewinski, T. (1992). Homogenizing stiffnesses of plates with periodic structure .*Int. J. Solids Structures*. Vol. 29, No.3, pp. 309-326, 1992.

Pruchnicki, E. (2017b). An exact two-dimensional model for heterogeneous plates. Mathematic and Mechanic of solid, https://doi.org/10.1177/1081286517752544.

Pruchnicki, E. (2018a). On the homogenization of nonlinear shell. Mathematic and mathematic, soumise le 08 01 18.

Pruchnicki, E. (2018b). Some specific aspects of linear homogenization shell theory. Mathematic and Mechanic of solid, soumise le 01 02 18.

Schneider, P, Kienzler, R, and Bohm M. (2014) Modeling of consistent second-order plate theories for anisotropic materials. *Journal of Applied Mathematic and Mechanic*. 94(1–2): 21–42.