Homogenization of both linear and nonlinear highly heterogeneous plate for modelling growth of plant

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In biological systems, 'growth' describes the process by which a biological tissue increases (growth) or decreases (atrophy) its body mass or volume. Biological systems are composed of soft material and allow finite strain deformation. Within the framework of nonlinear elasticity, soft materials can be modeled with an incompressible or nearly incompressible hyper elastic constitutive law. We will consider the theory of [1], the geometric deformation tensor is decomposed as the product of a growth tensor (describing the local change of mass or volume) and an elastic tensor. Only the elastic part of the deformation tensor induces stresses. Plants are structure composed of beams, plates and shells (leaves, petal, ….). The internal structure of the microscopic heterogeneous microstructure is given in details in [2-5]. This is show that the transverse dimensions of the thin structure (plate, shell, beam….) is much bigger than the size of the heterogeneities. So, we homogenize first the microstructure heterogeneities by tridimensional homogenization and then we have to apply standard reduction techniques in order to get a model independent of the transverse dimension(s) of the thin structure [6-7]. However, we can find leaves of plant with wavy upper and lower faces of the leaves (modelized as a plate, see figure 1 and 2). The size of the wavy heterogeneities is of the same order of the plate thickness. The full theory can be found in [8-9].
Other works on growth theory can be found in [10-15]. Growth theory can obviously remove, and we obtain a modeling suitable for industrial classical heterogeneous structure. We have Abaqus, Ansys, Zset and Castem.

Work for this PhD thesis

It relies on numerical implementation and analysis of the result of the theory given previously. However, if numerical problem appears a bibliography study must ne made for solving it.

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Figure 1: Wavy leaves of plant

Figure 2: Wavy leave of water lily